

Pure and gravitational radiation

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The well-known treatment of asymptotically flat vacuum fields is adapted to pure radiation fields. In this approach we find a natural normalization of the radiation null vector. The energy balance at null infinity shows that the mass loss results from a linear superposition of the pure and the gravitational radiation parts. By transformation to Bondi-Sachs coordinates the Kinnersley photon rocket is found to be the only axisymmetric Robinson-Trautman pure radiation solution without gravitational radiation.

PACS numbers: 04.30.-w, 04.20 Jb

I. INTRODUCTION

Pure radiation or null dust fields are fields of massless radiation which is considered as the incoherent superposition of waves with random phases and different polarizations but with the same propagation direction. The radiation can arise from fields of different types, from electro-magnetic null fields, massless scalar fields, neutrino fields or from the high frequency limit of gravitational waves. The energy-momentum tensor of pure radiation is

$$T_{ab} = \eta n_a n_b, \quad n_a n^a = 0, \quad \eta > 0, \quad (1)$$

where n^a is the tangent vector of the null congruence the radiation propagates along and η denotes the radiation density. The form of (1) is the result of an averaging process, i. e. the equations of the originating fields (Maxwell equations or Weyl equation) need not to be satisfied. However, (1) can be derived from a variational principle [1].

The local conservation law $T^{ab}{}_{;b} = 0$ implies that the radiation propagates along geodesics

$$n^a{}_{;b} n^b \propto n^a. \quad (2)$$

The energy density η is not uniquely defined by the decomposition (1) of the energy-momentum tensor, (1) is unchanged under the following renormalization of n^a and η

$$\tilde{n}^a = f n^a \quad \text{and} \quad \tilde{\eta} = \eta f^{-2}. \quad (3)$$

This annoying arbitrariness in the choice of n^a even can't be avoided by an affine parametrization of n^a , $n^a{}_{;b} n^b = 0$, the vector n^a still might be multiplied e. g. by a constant factor.

Asymptotically flat pure radiation fields can describe the exterior of isolated radiating sources. The prototype of an asymptotically flat pure radiation field, the Vaidya solution [2], is often taken as the exterior field of spherical symmetric fluid distributions. More general pure radiation fields, e. g. the pure radiation solutions of Robinson-Trautman type, correspondingly could be models for the exterior fields of more general isolated sources. They are expected to contain not only pure but also gravitational radiation.

Gravitational radiation in asymptotically flat space-times can be treated in an elegant way by using Bondi-Sachs (BS) coordinates, and the field equations can be solved by series expansion in terms of the luminosity parameter [3], [4]. In the next section we will apply this method to the general axisymmetric asymptotically flat pure radiation field. In order to solve the field equations step by step in terms of power series as in the vacuum case, the propagation vector n^a of the null radiation is adapted to the luminosity distance by fixing n^a at \mathcal{J}^+ . We calculate the leading order of the radiation density η and the total amount of energy which is radiated away by the null fluid. The energy balance at \mathcal{J}^+ is studied.

In the third section the general results are applied to axisymmetric Robinson-Trautman pure radiation solutions. By means of a transformation to BS-coordinates in terms of power series we can fit the pure radiation field to the luminosity distance. The general expressions for essential quantities, Bondi mass, news function and the leading term of the pure radiation energy density are given and we will show that the photon rocket of Kinnersley is the only solution of this class which does not contain gravitational radiation.

Notation: All quantities related to BS-metrics are denoted by capital letters, e. g. coordinates X^a and metric components G_{ab} . The metrics have signature $(+, +, +, -)$.

II. ASYMPTOTICALLY FLAT PURE RADIATION FIELDS

BS-coordinates $[\Theta, \Phi, R, U]$ can be introduced in the region far from isolated sources near \mathcal{J}^+ in asymptotically flat space-times. For that radially light rays are considered. The corresponding null congruence is geodesic and affinely parametrized, expanding and non-rotating, such that the tangent vector t^a is a gradient,

$$t_a t^a = 0, \quad t^a{}_{;b} t^b = 0, \quad t^a{}_{;a} \neq 0 \quad t_a = -U_{,a}. \quad (4)$$

The function U is taken as null coordinate and the intersection points of the null geodesics with a null hypersurface $U=\text{const.}$ are parametrized by the spherical coordinates $0 \leq \Theta \leq \pi$ and $0 \leq \Phi \leq 2\pi$. The distance between such an intersection point and the source is measured by the luminosity distance R which is normalized such that the closed two-surfaces $R=\text{const.}$, $U=\text{const.}$ have the surface of a sphere with the radius R . Near \mathcal{J}^+ , that is for sufficiently large R , $R > R_0$, the line-element in BS-coordinates has the form

$$ds^2 = R^2 h_{AB} (dX^A - W^A dU) (dX^B - W^B dU) - 2e^{2\beta} dU dR + \frac{V}{R} e^{2\beta} dU^2, \quad (5)$$

with

$$2h_{AB} dX^A dX^B = (e^{2\gamma} + e^{2\delta}) d\Theta^2 + 4 \sin \Theta \sinh(\gamma - \delta) d\Theta d\Phi + \sin^2 \Theta (e^{-2\gamma} + e^{-2\delta}) d\Phi^2. \quad (6)$$

Here β, γ, δ, V und W^A , $A, B \in \{1, 2\}$, are functions of the coordinates Θ , R and U , because of the axial symmetry they are independent of Φ , $G_{ab, \Phi} = 0$. The asymptotical behaviour of the metric (5) is given by the boundary conditions. The metric functions can be expanded in powers of $1/R$ in the coordinate range $0 \leq \Theta \leq \pi$, $0 \leq \Phi \leq 2\pi$, $U_1 < U < U_2$ and $R > R_0$,

$$\lim_{R \rightarrow \infty} \frac{V}{R} = -1 \quad \text{and} \quad \lim_{R \rightarrow \infty} R W^A = \lim_{R \rightarrow \infty} \beta = \lim_{R \rightarrow \infty} \gamma = \lim_{R \rightarrow \infty} \delta = 0 \quad (7)$$

hold. The line element (5) and the boundary conditions (7) are conserved under transformations of the Bondi-Metzner-Sachs (BMS) group [3], [4].

We are going to consider space-times containing outgoing pure radiation. In general, the geodesic null congruence the pure radiation is propagated along is not the null congruence (4) of the coordinate system. In particular, the radiation geodesics may have twist. But the asymptotic behaviour of these null congruences is the same, both end up at \mathcal{J}^+ (see figure 1). The null vector t^a has only a radial component, $t^a \rightarrow (0, 0, 1, 0)$ for $R \rightarrow \infty$. The tangent vector n^a of the radiation null geodesics is also dominated by its radial component, we choose the normalization factor such that $n^a \rightarrow (0, 0, 1, 0)$ for $R \rightarrow \infty$ holds too,

$$n^a = t^a + \mathcal{O}\left(\frac{1}{R}\right). \quad (8)$$

With this normalization the propagation of the pure radiation is fitted to the luminosity distance and the leading term of the radiation density η is fixed.

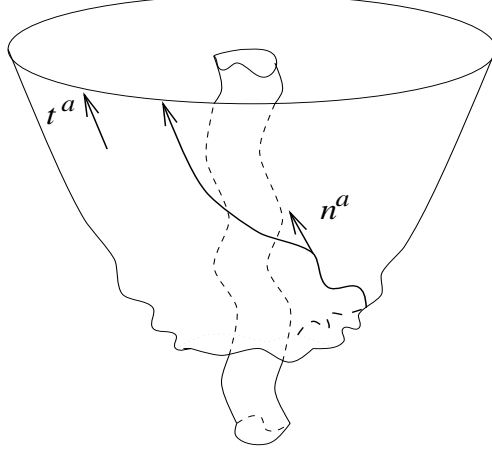


FIG. 1. Asymptotically flat field with pure radiation

Now we will solve the field equations in terms of power series of $1/R$ as it has been done in the vacuum case. For relevant results the ansatz

$$\beta = \frac{\beta_1}{R} + \frac{\beta_2}{R^2} + \frac{\beta_3}{R^3} + \frac{\beta_4}{R^4} + \mathcal{O}\left(\frac{1}{R^5}\right) \quad (9)$$

$$\gamma = \frac{\gamma_1}{R} + \frac{\gamma_2}{R^2} + \frac{\gamma_3}{R^3} + \mathcal{O}\left(\frac{1}{R^4}\right) \quad (10)$$

$$\delta = \frac{\delta_1}{R} + \frac{\delta_2}{R^2} + \frac{\delta_3}{R^3} + \mathcal{O}\left(\frac{1}{R^4}\right) \quad (11)$$

$$V = -R + 2M + \frac{q}{R} + \mathcal{O}\left(\frac{1}{R^2}\right) \quad (12)$$

$$W^1 = \frac{A_2}{R^2} + \frac{A_3}{R^3} + \frac{A_4}{R^4} + \mathcal{O}\left(\frac{1}{R^5}\right) \quad (13)$$

$$W^2 = \frac{B_2}{R^2} + \frac{B_3}{R^3} + \frac{B_4}{R^4} + \mathcal{O}\left(\frac{1}{R^5}\right), \quad (14)$$

with

$$A_3 = 2Q + \gamma_{1,\Theta}\delta_1 + \delta_{1,\Theta}\gamma_1 + \frac{1}{2}(\gamma_{1,\Theta}\gamma_1 + \delta_{1,\Theta}\delta_1) + 4\cot\Theta\gamma_1\delta_1 \quad (15)$$

$$B_3 = 2S + \gamma_{1,\Theta}\gamma_1 - \delta_{1,\Theta}\delta_1 + \cot\Theta(\gamma_1^2 - \delta_1^2). \quad (16)$$

for the metric functions is sufficient. The regularity at the axis demands γ , δ , $\gamma_{,\Theta}$ and $\delta_{,\Theta}$ to vanish for $\Theta = 0$ and $\Theta = \pi$. Additionally, the ansatz

$$n^1 = \frac{n^{11}}{R} + \frac{n^{12}}{R^2} + \frac{n^{13}}{R^3} + \mathcal{O}\left(\frac{1}{R^4}\right) \quad (17)$$

$$n^2 = \frac{n^{21}}{R} + \frac{n^{22}}{R^2} + \frac{n^{23}}{R^3} + \mathcal{O}\left(\frac{1}{R^4}\right) \quad (18)$$

$$n^3 = 1 + \frac{n^{31}}{R} + \frac{n^{32}}{R^2} + \frac{n^{33}}{R^3} + \mathcal{O}\left(\frac{1}{R^4}\right) \quad (19)$$

$$n^4 = \frac{n^{41}}{R} + \frac{n^{42}}{R^2} + \frac{n^{43}}{R^3} + \mathcal{O}\left(\frac{1}{R^4}\right) \quad (20)$$

for the contravariant components of the propagation vector $n^a = (n^1, n^2, n^3, n^4)$ is made, which corresponds to the normalization (8). The condition $n^a n_a = 0$ for n^a being null is expanded in powers of $1/R$ and the coefficients are set

equal to 0. This is the way all equations are solved. We prefer to describe the following steps rather than to present the explicit calculations which can easily be reproduced from these expansions. We find

$$n^{11} = n^{21} = n^{41} = 0 \quad (21)$$

and the U -component n^4 of n^a is fixed

$$n^{42} = \frac{(n^{12})^2}{2} + \sin^2 \Theta \frac{(n^{22})^2}{2}, \quad n^{43} = \dots \quad (22)$$

One of the ten field equations $R_{ab} = \kappa_0 \eta n_a n_b$ determines the radiation density η , the nine others are to be solved. Starting with R_{44} , we have

$$\frac{R_{44}}{n_4^2} = \mathcal{O}\left(\frac{1}{R^2}\right), \quad (23)$$

i. e. coefficients of powers of $1/R$ lower than 2 have to vanish. So from $R_{33}n_3^{-2} = \mathcal{O}(1/R^2)$ we get

$$\beta_1 = 0, \quad \beta_2 = -\frac{\gamma_1^2 + \delta_1^2}{8}, \quad \beta_3 = 0. \quad (24)$$

The corresponding conditions for $R_{13}/n_1 n_3$ and $R_{23}/n_1 n_3$ yield

$$A_2 = -\frac{\gamma_{1,\Theta} + \delta_{1,\Theta}}{2} - \cot \Theta (\gamma_1 + \delta_1) \quad \text{and} \quad \sin \Theta B_2 = -\frac{\gamma_{1,\Theta} - \delta_{1,\Theta}}{2} - \cot \Theta (\gamma_1 - \delta_1). \quad (25)$$

The coefficients of $1/R$ in $R_{11}n_1^{-2}$ and $R_{12}/n_1 n_3$ vanish for $\gamma_{2,U} = 0 = \delta_{2,U}$ and with the help of the freedom in the BMS-group we can put

$$\gamma_2 = 0 = \delta_2. \quad (26)$$

From the equation $R_{44}/n_4^2 = \kappa_0 \eta$ the leading term of the radiation density is calculated

$$\kappa_0 \eta = \frac{1}{R^2} \left[-2M_{,U} - \gamma_{1,U}^2 - \delta_{1,U}^2 - \frac{1}{2 \sin \Theta} \left[\frac{1}{\sin \Theta} (\sin^2 \Theta [\gamma_1 + \delta_1]_{,U})_{,\Theta} \right] \right] + \mathcal{O}\left(\frac{1}{R^3}\right). \quad (27)$$

For the remaining nine quantities $R_{ab}/n_a n_b$ (no summation) the $1/R^2$ -term is set equal to $\kappa_0 \eta$. Only eight of these nine equations are independent (the ninth equation corresponds to the null condition for the function n^{42}), they give us a system of algebraical equations for the functions q , A_4 , B_4 , β_4 and the U -derivatives $Q_{,U}$, $S_{,U}$, $\gamma_{3,U}$ and $\delta_{3,U}$. This result has to be interpreted in analogy to the vacuum case. The system is determined (up to the treated order of $1/R$) by the functions M , S , Q , γ_3 and δ_3 , given on a hypersurface $U=\text{const.}$, and by the functions γ_1 and δ_1 and the radiation density η , which must be given for all U . Note that the temporal development of the mass aspect M is determined not only by the news functions $\gamma_{1,U}$ and $\delta_{1,U}$ but also by the energy density η of the emitted pure radiation.

This fact becomes even clearer if the energy balance of the system at \mathcal{J}^+ is taken into consideration. The total amount \mathcal{E} of emitted pure radiation energy is calculated by integrating the density η over the closed surface $R=\text{const.}$, $U=\text{const.}$ in the limit $R \rightarrow \infty$. With the surface element $R^2 \sin \Theta d\Theta d\Phi + \mathcal{O}(R)$ it follows

$$\mathcal{E} = \lim_{R \rightarrow \infty} 2\pi \int_0^\pi \eta R^2 \sin \Theta d\Theta. \quad (28)$$

Introducing the notations $\langle f \rangle$ for the average of f over the intervall $0 \leq \Theta \leq \pi$ and $\mathcal{M} = \langle M \rangle$ for the Bondi mass, we get from (27) with the regularity properties of γ_1 and δ_1 and the convention $\kappa_0 = 8\pi$

$$\mathcal{M}_{,U} = -\mathcal{E} - \frac{1}{2} \langle \gamma_{1,U}^2 + \delta_{1,U}^2 \rangle. \quad (29)$$

The mass loss results from a linear superposition of pure and gravitational radiation, provided that the propagation vector of the pure radiation is normalized such that (8) holds. The corresponding result follows for the linear momentum. So for asymptotically flat pure radiation fields the decomposition of the energy-momentum tensor into propagation vector n^a and energy density η is not arbitrary, the normalization (8) is the natural relation between the physical and the geometrical fields n^a and t^a .

III. ROBINSON-TRAUTMAN PURE RADIATION SOLUTIONS

Robinson-Trautman space-times admit a shearfree, expanding and non-twisting congruence of null geodesics, which is a multiple eigen congruence of the Weyl tensor. The field equations for aligned pure radiation fields of this class are completely solved (see [5] §24.3). In the coordinates $[\vartheta, \varphi, r, u]$ the line element reads

$$ds^2 = \frac{r^2}{P^2} (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) - 2dudr + 2Hdu^2, \quad (30)$$

where for the axisymmetric case

$$P = P(\vartheta, u) \quad \text{and} \quad H = r(\ln P)_{,u} - \frac{K}{2} + \frac{m(u)}{r} \quad (31)$$

hold. K/r^2 denotes the Gaussian curvature of the surfaces $r=\text{const.}$, $u=\text{const.}$, which we assume to be distorted spheres,

$$K = P^2 [(\ln P)_{,\vartheta\vartheta} + \cot \vartheta (\ln P)_{,\vartheta} + 1] =: \Delta \ln P. \quad (32)$$

The vector $k^a = (0, 0, 1, 0)$ is the tangent vector of the affinely parametrized null geodesics, if k^a is chosen as propagation vector of the pure radiation, $R_{ab} = \kappa_0 \eta k_a k_b$, the radiation density reads

$$\kappa_0 \eta = \frac{2}{r^2} \left[-m_{,u} + 3m(\ln P)_{,u} + \frac{1}{4} \Delta \Delta \ln P - \frac{P^2}{4} \right]. \quad (33)$$

If we put $P = 1$ we arrive at the Vaidya solution for which the coordinates $[\vartheta, \varphi, r, u]$ are BS-coordinates and by comparison with the components of the BS-metric (5) and the ansatz (9)-(14) we find $\mathcal{M} = m$ and $\gamma_1 = \delta_1 = 0$.

Now we are going to the general line element (30) and transform it into BS-coordinates. The procedure is similar to that presented in [6], the BS-coordinates are expanded in power series in terms of $1/r$. In the axisymmetric non-rotating case we set

$$\Phi = \varphi. \quad (34)$$

By the line element of the two-surfaces $r=\text{const.}$, $u=\text{const.}$ the ansatz $R \sim r/P$ is suggested, such that we start with

$$\Theta = T_0 + \frac{T_1}{r} + \frac{T_2}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right), \quad (35)$$

$$R = \frac{r}{P} + R_0 + \frac{R_1}{r} + \mathcal{O}\left(\frac{1}{r^2}\right), \quad (36)$$

$$U = U_0 + \frac{U_1}{r} + \frac{U_2}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right), \quad (37)$$

where all coefficients depend on ϑ and u . For the determination of these coefficients we step by step make use of the properties

$$G^{44} = 0 = G^{14}, \quad G^{34} = -1 + \mathcal{O}\left(\frac{1}{R^2}\right), \quad G^{13} = \mathcal{O}\left(\frac{1}{R^2}\right), \quad G^{11}G^{22} = (\sin^2 \Theta R^4)^{-1}, \quad (38)$$

the transformed contravariant metric components

$$G^{ab} = \frac{\partial X^a}{\partial x^i} \frac{\partial X^b}{\partial x^j} g^{ij} \quad (39)$$

must have. The expansion of G^{13} yields

$$G^{13} = -\frac{T_{0,u}}{P} + \mathcal{O}\left(\frac{1}{r}\right), \quad (40)$$

from which $T_{0,u} = 0$ follows. Using the freedom in the BMS-group we can set

$$T_0 = \vartheta \quad (41)$$

for simplicity. From

$$G^{34} = -\frac{U_{0,u}}{P} + \mathcal{O}\left(\frac{1}{r}\right) \quad (42)$$

we have

$$U_{0,u} = P, \quad (43)$$

which fixes the u -dependence of U_0 , the remaining freedom in the ϑ -dependence also corresponds to a transformation in the BMS-group. All higher coefficients can now be expressed in terms of $T_0 = \vartheta$ and U_0 . The condition $G^{44} = 0$ yields

$$U_1 = -\frac{1}{2}PU_{0,\vartheta}^2 \quad \text{and} \quad U_2 = \frac{1}{2}P^2U_{0,\vartheta}^2U_{0,\vartheta\vartheta} \quad (44)$$

and from $G^{14} = 0$ we get

$$T_1 = -PU_{0,\vartheta} \quad \text{and} \quad T_2 = P^2U_{0,\vartheta}U_{0,\vartheta\vartheta}. \quad (45)$$

Finally the coefficients in the expansion of R are to be calculated from $G^{11}G^{22} = (\sin^2 \Theta R^4)^{-1}$, the leading term of that condition confirms the ansatz $R \sim r/P$ and the higher terms give

$$R_0 = \frac{1}{2}[U_{0,\vartheta\vartheta} + \cot \vartheta U_{0,\vartheta}] \quad (46)$$

and

$$R_1 = \frac{U_{0,\vartheta}^2}{4}[P_{,\vartheta\vartheta} - \cot \vartheta P_{,\vartheta}] - \frac{P}{8}[U_{0,\vartheta\vartheta} + \cot \vartheta U_{0,\vartheta}]^2 + \frac{PU_{0,\vartheta}}{2}[U_{0,\vartheta\vartheta\vartheta} - \cot \vartheta U_{0,\vartheta\vartheta}]. \quad (47)$$

Thus the coefficients in the ansatz (35)-(37) are determined, terms of higher order can be calculated in the same manner by a continued expansion of the relevant conditions.

The ansatz (35)-(37) is sufficient to extract characteristic quantities from the metric components. From (5) and (12) we know the component G^{33} and its series expansion in terms of $1/R$ which can be converted into a power series of $1/r$ by means of (36)

$$G^{33} = -\frac{V}{R}e^{-2\beta} = 1 - 2\frac{M}{R} + \mathcal{O}\left(\frac{1}{R^2}\right) = 1 - 2\frac{MP}{r} + \mathcal{O}\left(\frac{1}{r^2}\right). \quad (48)$$

Now the mass aspect M can be read off from the $1/r$ -expansion of G^{33} as a function of ϑ and u

$$M = \frac{m}{P^3} + \frac{U_{0,\vartheta}^2 B_{,u}}{4P} - \frac{U_{0,\vartheta\vartheta} B}{4} - \frac{U_{0,\vartheta}}{4}[2B_{,\vartheta} + 3 \cot \vartheta B], \quad (49)$$

where B is defined by

$$B = \frac{\sin \vartheta}{P} \left(\frac{P_{,\vartheta}}{\sin \vartheta} \right)_{,\vartheta}. \quad (50)$$

In the non-rotating case $\gamma = \delta$ and $W^2 = 0$ hold for the metric functions, such that the news is given by γ_1 . As the mass aspect M , the function γ_1 is calculated from the transformed metric

$$G^{34} = -e^{-2\beta} = -1 - \frac{\gamma_1^2 + \delta_1^2}{4R^2} + \mathcal{O}\left(\frac{1}{R^4}\right) = -1 - \frac{\gamma_1^2 P^2}{2r^2} + \mathcal{O}\left(\frac{1}{r^3}\right) \quad (51)$$

as

$$\gamma_1^2 = \frac{\sin^2 \vartheta}{4} \left(\frac{U_{0,\vartheta}}{\sin \vartheta} \right)_{,\vartheta}^2. \quad (52)$$

By differentiation with respect to U , which can be calculated as

$$\gamma_{1,u} = \gamma_{1,U} \lim_{r \rightarrow \infty} \frac{\partial U}{\partial u} \quad (53)$$

we get

$$\gamma_{1,U}^2 = \frac{B^2}{4} = \frac{\sin^2 \vartheta}{4P^2} \left(\frac{P_{,\vartheta}}{\sin \vartheta} \right)_{,\vartheta}^2. \quad (54)$$

Finally the propagation of the pure radiation is adapted to the luminosity distance R , i. e. the radiation null vector n^a should be normalized such that (8) holds. The transformed null tetrad $k^{a'}$

$$k^{a'} = \frac{\partial X^{a'}}{\partial r} = \left(\mathcal{O}\left(\frac{1}{r^2}\right), 0, \frac{1}{P} + \mathcal{O}\left(\frac{1}{r^2}\right), \mathcal{O}\left(\frac{1}{r^2}\right) \right), \quad (55)$$

shows that k^a does not fulfil the normalization condition (8). Moreover, the normalized propagation vector is given by

$$n^a = k^a \left[P + \mathcal{O}\left(\frac{1}{r}\right) \right] \quad (56)$$

and the leading term of the correspondingly normalized energy density of the pure radiation reads

$$\begin{aligned} \eta &= \frac{1}{4\pi r^2} \left[-\frac{m_{,u}}{P^2} + 3m \frac{P_{,u}}{P^3} + \frac{1}{4P^2} \Delta \Delta \ln P - \frac{1}{4} \right] + \mathcal{O}\left(\frac{1}{r^3}\right) \\ &= \frac{1}{4\pi R^2} \left[-\frac{1}{P} \left(\frac{m}{P^3} \right)_{,u} + \frac{1}{4P^4} \Delta \Delta \ln P - \frac{1}{4P^2} \right] + \mathcal{O}\left(\frac{1}{R^3}\right). \end{aligned} \quad (57)$$

IV. THE PHOTON ROCKET

Recently the 'photon rocket' solution of Kinnersley [7] has been the subject of articles where it is proven not to contain gravitational radiation [8] - [10]. With the results of the foregoing sections we are now able (a) to give another proof of this fact and (b) to show that the photon rocket is the only axisymmetric Robinson-Trautman solution with pure and without gravitational radiation.

Kinnersley's solution is interpreted as the field of a particle emitting pure radiation anisotropically, and accelerating because of the recoil. In the coordinates $[\tilde{\vartheta}, \varphi, r, u]$ the line element reads

$$ds^2 = r^2 ([d\tilde{\vartheta} + a \sin \tilde{\vartheta} du]^2 + \sin^2 \tilde{\vartheta} d\varphi^2) - 2dudr - \left(1 - 2ar \cos \tilde{\vartheta} - \frac{2m}{r} \right) du^2, \quad (58)$$

where the functions m and a depend on the retarded time u and can be interpreted as the mass and the acceleration of the particle, respectively. Introducing a new coordinate ϑ by

$$\tan \frac{\vartheta}{2} = e^A \tan \frac{\tilde{\vartheta}}{2}, \quad A = A(u), \quad A_{,u} = a, \quad (59)$$

the line element (58) is cast into Robinson-Trautman form with the metric function

$$P = \cosh A + \sinh A \cos \vartheta. \quad (60)$$

If this function P is put into the formula for the news function (54) we get $\gamma_{1,U} = 0$, i. e. there is no gravitational radiation. Conversely, solving the equation for vanishing news, we get $P = b(u) + c(u) \cos \vartheta$ which can be put into the form (60) by a coordinate transformation which preserves the form of the Robinson-Trautman line element.

The result that the Kinnersley rocket is the only pure radiation solution without gravitational radiation is in contradiction to [11] where the energy balance for a perturbed Kinnersley metric is calculated. But it confirms the hypothesis in [9] that more anisotropic pure radiation solutions than the photon rocket would emit gravitational radiation.

V. CONCLUSION

We have been studying asymptotically flat pure radiation fields in terms of Bondi-Sachs coordinates. In this framework we find a natural normalization for the propagation vector of the pure radiation. The energy balance at \mathcal{J}^+ shows a linear superposition of pure and gravitational radiation. The application to the Robinson-Trautman pure radiation solutions confirms that the Bondi-Sachs coordinates are powerful in the treatment of asymptotically flat fields, not only for vacuum but also for pure radiation fields.

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